Answer to examples and exercises in lecture 08 - Schedulability part 3

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$$w_i^{n+1} = e_i + \sum_{j=1}^{k} \left\lceil \frac{w_i^n}{p_j} \right\rceil \cdot C_j + \sum_{r=1}^{R} u(r,i) \cdot C_s(r)$$

Example 1 (Apply Priority Inheritance)

Task	e	p	D	P
1	3	25	7	1
2	2	12	-	2
3	5	17	-	3
4	6	24	-	4

Resource	$C_s(r)$	Used by
1	2	1,3,4
2	4	2,4

$B_{T1} = Max(u(r1,T1) \cdot C_s(r1) + u(r2,T1) \cdot C_s(r_2)) = 1 * 2 + 0 * 4 = 2$

$R_{T1} = e_{T1} + B_{T1} = 2 + 3 = 5 <= 7$ therefore $T1$ meets its deadline.

$B_{T2} = u(r1,T2) \cdot C_s(r1) + u(r2,T2) \cdot C_s(r_2) = 1 * 2 + 1 * 4 = 6$

$W_{T2}^1 = 2 + \lceil \frac{2}{25} \rceil \cdot 3 + B_{T2} = 2 + 1 \cdot 3 + 6 = 11$

$W_{T2}^2 = 2 + \lceil \frac{11}{25} \rceil \cdot 3 + B_{T2} = 2 + 1 \cdot 3 + 6 = 11$

$R_(T2) = 11 <= 12$ therefore $T2$ meets its deadline.

$B_{T3} = u(r1,T3) \cdot C_s(r1) + u(r2,T3) \cdot C_s(r_2) = 1 * 2 + 1 * 4 = 6$

$W_{T3}^1 = 5 + \lceil \frac{5}{25} \rceil \cdot 3 + \lceil \frac{5}{12} \rceil \cdot 2 + B_{T3} = 5 + 1 \cdot 3 + 1 \cdot 2 + 6 = 5 + 3 + 2 + 6 = 16$

$W_{T3}^2 = 5 + \lceil \frac{16}{25} \rceil \cdot 3 + \lceil \frac{16}{12} \rceil \cdot 2 + B_{T3} = 5 + 1 \cdot 3 + 2 \cdot 2 + 6 = 5 + 3 + 4 + 6 = 18$

$18 >= 17$ therefore $T3$ does meets its deadline. The tasks are not schedulable.

Example 2 (Apply Ceiling Priority Protocols, tasks and resources are same as example 1)

$$w_i^{n+1} = e_i + \sum_{j=1}^{k} \left\lceil \frac{w_i^n}{p_j} \right\rceil \cdot C_j + Max_{r=1}^{R} u(r,i) \cdot C_s(r)$$

$B_{T1} = Max\big(u(r1,T1) \cdot C_s(r1), u(r2,T1) \cdot C_s(r_2)\big) = Max\big(1*2, 0*4\big) = 2$

$R_{T1} = e_{T1} + B_{T1} = 2 + 3 = 5 <= 7$ therefore $T1$ meets its deadline.

$B_{T2} = Max\big(u(r1,T2) \cdot C_s(r1), u(r2,T2) \cdot C_s(r_2)\big) = Max\big(1*2, 1*4\big) = 4$

$W_{T2}^1 = 2 + \lceil \frac{2}{25} \rceil \cdot 3 + B_{T2} = 2 + 1 \cdot 3 + 4 = 9$

$W_{T2}^2 = 2 + \lceil \frac{9}{25} \rceil \cdot 3 + B_{T2} = 2 + 1 \cdot 3 + 4 = 9$

$R_{T2} = 9 <= 12$ therefore $T2$ meets its deadline.

$B_{T3} = Max\big(u(r1,T3) \cdot C_s(r1), u(r2,T3) \cdot C_s(r_2)\big) = Max\big(1*2, 1*4\big) = 4$

$W_{T3}^1 = 5 + \lceil \frac{5}{25} \rceil \cdot 3 + \lceil \frac{5}{12} \rceil \cdot 2 + B_{T3} = 5 + 1 \cdot 3 + 1 \cdot 2 + 6 = 5 + 3 + 2 + 4 = 14$

$W_{T3}^2 = 5 + \lceil \frac{14}{25} \rceil \cdot 3 + \lceil \frac{14}{12} \rceil \cdot 2 + B_{T3} = 5 + 1 \cdot 3 + 2 \cdot 2 + 6 = 5 + 3 + 4 + 4 = 16$

$W_{T3}^3 = 5 + \lceil \frac{16}{25} \rceil \cdot 3 + \lceil \frac{16}{12} \rceil \cdot 2 + B_{T3} = 5 + 1 \cdot 3 + 2 \cdot 2 + 6 = 5 + 3 + 4 + 4 = 16$

$R_{T3} = 16 <= 17$ therefore $T3$ meets its deadline.

$B_{T4} = 0$

$W_{T4}^1 = 6 + \lceil \frac{6}{25} \rceil \cdot 3 + \lceil \frac{6}{12} \rceil \cdot 2 + \lceil \frac{6}{17} \rceil \cdot 5 + B_{T4} = 6 + 1 \cdot 3 + 1 \cdot 2 + 1 \cdot 5 = 6 + 3 + 2 + 5 + 0 = 16$

$W_{T4}^2 = 6 + \lceil \frac{16}{25} \rceil \cdot 3 + \lceil \frac{16}{12} \rceil \cdot 2 + \lceil \frac{16}{17} \rceil \cdot 5 + B_{T4} = 6 + 1 \cdot 3 + 2 \cdot 2 + 1 \cdot 5 = 6 + 3 + 4 + 5 + 0 = 18$

$W_{T4}^3 = 6 + \lceil \frac{18}{25} \rceil \cdot 3 + \lceil \frac{18}{12} \rceil \cdot 2 + \lceil \frac{18}{17} \rceil \cdot 5 + B_{T4} = 6 + 1 \cdot 3 + 2 \cdot 2 + 2 \cdot 5 = 6 + 3 + 4 + 10 + 0 = 23$

$W_{T4}^4 = 6 + \lceil \frac{23}{25} \rceil \cdot 3 + \lceil \frac{23}{12)} \rceil \cdot 2 + \lceil \frac{23}{17} \rceil \cdot 5 + B_{T4} = 6 + 1 \cdot 3 + 2 \cdot 2 + 2 \cdot 5 = 6 + 3 + 4 + 10 + 0 = 23$

$R_{T4} = 23 <= 24$ therefore $T3$ meets its deadline.